CH:4 PAIR OF STRAIGHT LINES

★ **Equations of line in different forms:**

	Name	Given	Equation
1	Two point form	$A(x_1, y_1)$, $B(x_2, y_2)$	$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$
2	Slope-point form	slope $= m$, $A(x_1, y_1)$	$y - y_1 = m(x - x_1)$
3	Slope-Intercept form	slope $= m$, y-intercept $= c$	$y = mx + c$
4	Double intercept form	x-intercept $= a$, y-intercept $= b$	$\frac{x}{a} + \frac{y}{b} = 1$
5	Normal form	p = length of normal from origin, α = Inclination of normal	$x\cos\alpha + y\sin\alpha = p$
6	General form	—	$ax + by + c = 0$

★ For the line $ax + by + c = 0$

slope$(m) = -\frac{a}{b}$, x-intercept $= -\frac{c}{a}$,

y-intercept $= -\frac{c}{b}$

★ If m_1 & m_2 are slopes of two lines & if θ is acute angle between them then

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

- Lines are perpendicular if $m_1 \cdot m_2 = -1$
- Lines are parallel if $m_1 = m_2$.

★ If $U \equiv a_1 x + b_1 y + c_1 = 0$ & $V \equiv a_2 x + b_2 y + c_2 = 0$ are two lines then

(1) $U \parallel V$ if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

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② $u \perp v$ if $a_1a_2 + b_1b_2 = 0$.

★ If u & v are two lines then. $uv = 0$ represents combined equation of two lines.

★ $ax^2 + 2hxy + by^2 = 0$ is the homogeneous equation of pair of lines thro' origin.

★ Acute angle θ between the lines $ax^2 + 2hxy + by^2 = 0$ is

$$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

★ The roots of above eqn are m_1 & m_2.

$$m_1 + m_2 = -\frac{2h}{b} \quad \& \quad m_1 m_2 = \frac{a}{b}$$

★ Lines are perpendicular when

$$m_1 m_2 = -1$$

ie. $a + b = 0$

★ Lines are parallel or coincident when

$m_1 = m_2$ ie. $m_1 - m_2 = 0$

ie. $h^2 = ab$

★ General equation of second degree of lines is

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

★ Distance of $P(x_1, y_1)$ from the line $ax + by + c = 0$ is

$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|.$$

★ Nature of lines:

1) If $h^2 - ab > 0$, lines are real & distinct.
2) If $h^2 - ab = 0$, lines are real & coincident
3) If $h^2 - ab < 0$, lines are not real. Eqn does not represent lines.

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★ The general eq^n $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of lines if

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

i.e. $abc+2fgh-af^2-bg^2-ch^2=0$.

- Acute angle between the lines is -

$$\tan\theta = \left| \frac{2\sqrt{h^2-ab}}{a+b} \right|$$

- ~~Lines~~ are perpendicular if $a+b=0$.

Lines are parallel if $h^2-ab=0$

★ Joint eq^n of bisectors of angle betwen two lines $ax^2+2hxy+by^2=0$ is

$$\frac{x^2-y^2}{a-b} = \frac{xy}{h}$$

★ Distance between parallel lines: $ax+by+c_1=0$ & $ax+by+c_2=0$ $d = \frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$

① Joint equation of co-ordinate axes, in a plane is ___

a) $x^2-y^2=0$ b) $x^2+y^2=1$

c) $xy=0$ d) $xy=x+y$

⇒ (c) Eq^n of x-axis : $y=0$, y-axis : $x=0$

∴ Joint equation is : $xy=0$

② Joint equation of two lines, through the origin, having slopes 2 & -2 is ___

a) $x^2-4y^2=0$ b) $4x^2-y^2=0$

c) $x^2-2y^2=0$ d) $2x^2-y^2=0$

⇒ (b) Lines are passing thro' origin.

∴ Eq^n is $y=mx$

∴ $y=2x$ & $y=-2x$

∴ $2x-y=0$ & $2x+y=0$

$| \quad \frac{}{a+b} |$

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∴ Joint equation: $(2x-y)(2x+y)=0$
$\therefore 4x^2-y^2=0$

(3) Joint equation of two lines, through the origin, such that one of them is parallel & the other perpendicular to line $2x+3y+c=0$ is ___

a) $6x^2-5xy-6y^2=0$ b) $6x^2-5xy+6y^2=0$ (2)
c) $6x^2+5xy-6y^2=0$ d) $6x^2+5xy+6y^2=0$

⇒ (c) Slope of line $2x+3y+c=0$
is $-\frac{a}{b} = -\frac{2}{3}$

∴ For parallel line, $m_1 = -\frac{2}{3}$ $\therefore y = -\frac{2}{3}x$
$\therefore 3y = -2x$
$\therefore 2x+3y=0$

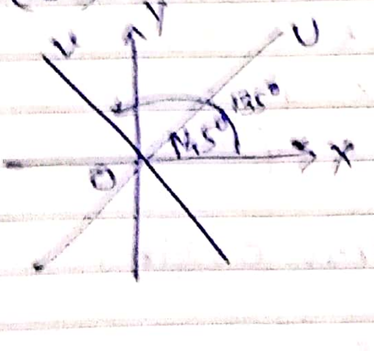
For ⊥ line, $m_2 = \frac{3}{2}$ $\therefore y = \frac{3}{2}x$
$\therefore 2y = 3x$
$\therefore 3x-2y=0$

∴ Joint equation is
$(2x+3y)(3x-2y)=0$
$\therefore 6x^2-4xy+9xy-6y^2=0$
$\therefore 6x^2+5xy-6y^2=0$

(4) Joint equation of lines bisecting angles between co-ordinate axes is ___

a) $x^2+y^2=0$ b) $x^2-y^2=0$
c) $x^2-2y^2=0$ d) $x^2+y^2=1$ (5)

⇒ (b)



(1) For line u,
Slope $m_1 = \tan 45^\circ = 1$
∴ Eq^n is: $y = m_1 x$ $\therefore y = x$
$\therefore x - y = 0$

(2) For line v,
Slope $m_2 = \tan 135^\circ$
$= \tan(180^\circ - 45^\circ)$
$= -\tan 45^\circ$
$= -1$

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$\therefore$ Eq^n is: $y = m_2 x$

$\therefore y = -x$

$\therefore x + y = 0$

$\therefore$ Joint equation is

$(x-y)(x+y) = 0$

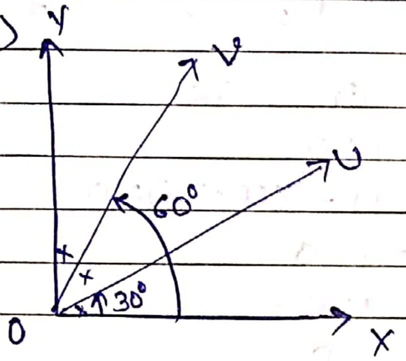
$\therefore x^2 - y^2 = 0$

5) Joint equation of lines, trisecting angles in first & third quadrant, is ___

a) $\sqrt{3}(x^2-y^2) - 4xy = 0$ b) $\sqrt{3}(x^2-y^2) + 4xy = 0$

c) $\sqrt{3}(x^2+y^2) + 4xy = 0$ d) $\sqrt{3}(x^2+y^2) - 4xy = 0$

$\Rightarrow$ (d)



① For line u: slope

$m_1 = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore$ Eq^n is: $y = \frac{1}{\sqrt{3}} x$

$\therefore \sqrt{3} y = x$

$\therefore x - \sqrt{3} y = 0$

② For line v:

slope $m_2 = \tan 60° = \sqrt{3}$

$\therefore$ Eq^n is: $y = \sqrt{3} x$

$\therefore \sqrt{3} x - y = 0$

$\therefore$ Joint equation is

$(x - \sqrt{3} y)(\sqrt{3} x - y) = 0$

$\therefore \sqrt{3} x^2 - xy - 3xy + \sqrt{3} y^2 = 0$

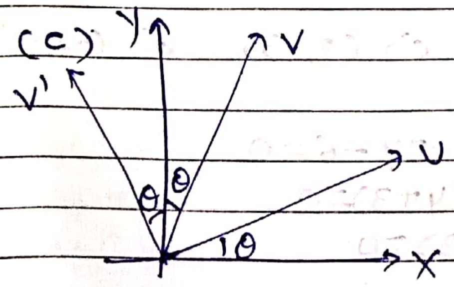
$\therefore \sqrt{3}(x^2 + y^2) - 4xy = 0$

6) If two lines $ax^2 + 2hxy + by^2 = 0$ are equally inclined with co-ordinate axes, then ___

a) $h = 0$ and $ab < 0$ b) $a = b$

c) $a = \pm b$ d) $a^2 + b^2 = 0$

$\Rightarrow$ (c)



① For line u:

Slope $= \tan\theta$

$\therefore m_1 = \tan\theta$

② For line v:

slope $= \tan(90 - \theta)$ or $\tan(90 + \theta)$

$a + b$

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$\therefore m_2 = +\cot\theta$ or $-\cot\theta$

$\therefore m_2 = \pm\cot\theta = \pm\frac{1}{\tan\theta} = \pm\frac{1}{m_1}$

$\therefore m_1 m_2 = \pm 1$

But $m_1 m_2 = \frac{a}{b}$ $\left\{\right.$ For line $ax^2+2hxy+by^2=0$

$\therefore \frac{a}{b} = \pm 1$

$\therefore a = \pm b$

(7) Lines jointly given by $x^2-9y^2-x+3y=0$ intersect each other in the point ___

a) $\left(-\frac{1}{2}, \frac{1}{6}\right)$ b) $\left(\frac{1}{2}, -\frac{1}{6}\right)$ c) $\left(\frac{1}{2}, \frac{1}{6}\right)$ d) $\left(\frac{1}{3}, \frac{2}{3}\right)$

$\Rightarrow$ (c) Joint equation of lines is -

$$x^2-9y^2-x+3y=0$$

$$\therefore (x+3y)(x-3y)-(x-3y)=0$$

$$\therefore (x-3y)(x+3y-1)=0$$

$\therefore$ Separate equations are -

$$x-3y=0 \quad \text{—— (1)}$$

$$x+3y-1=0 \quad \text{—— (2)}$$

Eq^n (1) - Eq^n (2) $\Rightarrow$

$$\begin{array}{r} x-3y=0 \\ -\ \ x+3y=1 \\ \hline -6y=-1 \end{array}$$

$\therefore y = 1/6$

Eq^n (1) $\Rightarrow$

$$x-3(1/6)=0$$

$$\therefore x-\frac{1}{2}=0 \qquad \therefore x = 1/2$$

$\therefore$ Point of intersection is $\left(\frac{1}{2}, \frac{1}{6}\right)$.

(8) Lines whose combined equation is $xy+3x-2y-6=0$ pass through the point ___

a) $(2,3)$ b) $(-2,3)$ c) $(2,-3)$ d) $(-2,-3)$

$\Rightarrow$ (c) Joint eq^n: $xy+3x-2y-6=0$

$$\therefore x(y+3)-2(y+3)=0$$

$$\therefore (x-2)(y+3)=0$$

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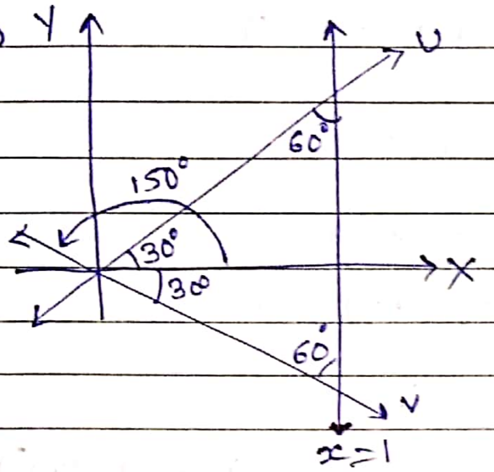
$\therefore x = 2$ & $y = -3$

$\therefore$ Point of intersection is $(2, -3)$.

9) Joint equation of lines, through the origin, making an equilateral triangle with the line $x = 1$, is ____

a) $3x^2 - y^2 = 0$ b) $x^2 - 3y^2 = 0$
c) $x^2 - \sqrt{3}y^2 = 0$ d) $3x^2 + y^2 = 1$

→ (b)



Lines make angle $30°$ & $150°$ with the positive side of X-axis.

1) For line U:

Slope $(m_1) = \tan 30°$

$\therefore m_1 = \frac{1}{\sqrt{3}}$

$\therefore$ Eqⁿ: $y = \frac{1}{\sqrt{3}}x$

$\therefore \sqrt{3}y = x$

$\therefore x - \sqrt{3}y = 0$

2) For line V: Slope $(m_2) = \tan 150°$

$= \tan(90° + 60°)$

$= -\cot 60° = -\frac{1}{\sqrt{3}}$

$\therefore$ Eqⁿ: $y = -\frac{1}{\sqrt{3}}x$

$\therefore \sqrt{3}y = -x$

$\therefore x + \sqrt{3}y = 0$

$\therefore$ Joint equation is: $(x - \sqrt{3}y)(x + \sqrt{3}y) = 0$

$\therefore x^2 - 3y^2 = 0$

10) Joint equation of two lines, through (x_1, y_1), perpendicular to two lines $ax^2 + 2hxy + by^2 = 0$ is ____

a) $b(x - x_1)^2 + 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$
b) $b(x - x_1)^2 - 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$
c) $a(x - x_1)^2 + 2h(x - x_1)(y - y_1) + b(y - y_1)^2 = 0$
d) $a(y_1 - y_2) - 2h(x - x_1)(y - y_1) + b(x - x_1) = 0$

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⇒ (b) Let m_1 & m_2 be slopes of 2 lines.

$$m_1 + m_2 = -\frac{2h}{b}$$

$$m_1 m_2 = \frac{a}{b}$$

∴ Slopes of reqd lines are $-\frac{1}{m_1}$ & $-\frac{1}{m_2}$.

∴ Eqⁿ ① $y - y_1 = -\frac{1}{m_1}(x - x_1)$

$$\therefore m_1(y - y_1) = -(x - x_1)$$

$$\therefore (x - x_1) + m_1(y - y_1) = 0$$

Eqⁿ ② ⇒ $y - y_1 = -\frac{1}{m_2}(x - x_1)$

$$\therefore m_2(y - y_1) = -(x - x_1)$$

$$\therefore (x - x_1) + m_2(y - y_1) = 0.$$

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∴ Joint eqⁿ:

$$[(x - x_1) + m_1(y - y_1)][(x - x_1) + m_2(y - y_1)] = 0$$

$$\therefore (x - x_1)^2 + m_2(x - x_1)(y - y_1) + m_1(x - x_1)(y - y_1) + m_1 m_2 (y - y_1)^2 = 0$$

$$\therefore (x - x_1)^2 + (m_1 + m_2)(x - x_1)(y - y_1) + m_1 m_2 (y - y_1)^2 = 0$$

$$\therefore (x - x_1)^2 - \frac{2h}{b}(x - x_1)(y - y_1) + \frac{a}{b}(y - y_1)^2 = 0$$

$$\therefore b(x - x_1)^2 - 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$$

Similarly, For parallel lines joint eqⁿ is:

$$a(x - x_1)^2 + 2h(x - x_1)(y - y_1) + b(y - y_1)^2 = 0$$

⑪ If one of the lines $6x^2 + cxy + y^2 = 0$ is $y + 2x = 0$, then $c =$ ____

a) -3 b) -4 c) -5 d) 5 13

⇒ (d) Comparing given eqⁿ with $ax^2 + 2hxy + by^2 = 0$, we get

$$a = 6,\ 2h = c,\ b = 1$$

$$\therefore h = \frac{c}{2}$$

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Slope of the line $y+2x=0$ ie. $2x+y=0$ is $-\frac{2}{1}=-2$ Let $m_1=-2$

Let m_2 be slope of another line.

$$m_1+m_2=-\frac{2h}{b}=\frac{-c}{1}=-c$$

$$\therefore -2+m_2=-c$$

$$\therefore m_2=2-c \quad \text{---} (1)$$

Also, $m_1 m_2=\frac{a}{b}$

$$\therefore -2(2-c)=\frac{6}{1} \quad \text{--- from (1)}$$

$$\therefore -4+2c=6$$

$$\therefore 2c=10 \qquad \therefore c=5$$

12) If slopes of lines $3x^2+kxy-y^2=0$ differ by 4, then $k=$ ___

a) -2 b) 2 c) ± 2 d) $\pm 2\sqrt{7}$

⇒ (c) Comparing with $ax^2+2hxy+by^2=0$, we get

$$a=3,\ 2h=k,\ b=-1$$

Also, $|m_1-m_2|=4$ --- Given

Now, $m_1+m_2=-\frac{2h}{b}=\frac{-k}{-1}=k$

& $m_1 m_2=\frac{a}{b}=\frac{3}{-1}=-3$

We know that,

$$(m_1-m_2)^2=(m_1+m_2)^2-4m_1m_2$$

$$\therefore (4)^2=(k)^2-4(-3)$$

$$\therefore 16=k^2+12$$

$$\therefore k^2=4 \qquad \therefore k=\pm 2$$

13) Measure of angle between the lines $3x^2-8xy-3y^2=0$ is ___

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

⇒ (a) Comparing with $ax^2+2hxy+by^2=0$, we get

$$a=3,\ 2h=-8,\ b=-3 \qquad \therefore h=-4$$

$$\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

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$$= \left| \frac{2\sqrt{(-4)^2 - (3)(-3)}}{3-3} \right|$$

$$= \left| \frac{2\sqrt{16-9}}{0} \right| = \infty$$

$$\therefore \theta = \frac{\pi}{2}$$

(14) If acute angle between lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{\pi}{3}$, then $(a+3b)\cdot(3a+b) =$ ____

a) h^2 b) $2h^2$ c) $3h^2$ d) $4h^2$

⇒ (d)

$$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

$$\therefore \tan\frac{\pi}{3} = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

$$\therefore \sqrt{3} = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right| \quad \text{squaring}$$

$$\therefore 3(a+b)^2 = 4(h^2 - ab)$$

$$\therefore 3(a^2 + 2ab + b^2) = 4h^2 - 4ab$$

$$\therefore 3a^2 + 6ab + 3b^2 = 4h^2 - 4ab$$

$$\therefore 3a^2 + 10ab + 3b^2 = 4h^2$$

$$\therefore 3a^2 + 9ab + ab + 3b^2 = 4h^2$$

$$\therefore 3a(a+3b) + b(a+3b) = 4h^2$$

$$\therefore (a+3b)\cdot(3a+b) = 4h^2$$

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(15) If ratio of slopes of lines $ax^2 + 2hxy + by^2 = 0$ is 1:3, then $h^2 : ab =$ ____

a) $\frac{1}{3}$ b) $\frac{3}{4}$ c) $\frac{4}{3}$ d) 1

⇒ (c) Ratio of slopes = 1 : 3

$$\therefore \frac{m_1}{m_2} = \frac{1}{3} \qquad \therefore m_2 = 3m_1$$

$$\therefore m_1 + m_2 = -\frac{2h}{b}$$

$$\therefore m_1 + 3m_1 = -\frac{2h}{b}$$

$$\therefore 4m_1 = -\frac{2h}{b}$$

$$\therefore m_1 = -\frac{h}{2b}$$

$$\therefore m_1^2 = \frac{h^2}{4b^2} \quad \text{— (1)}$$

$$m_1 m_2 = \frac{a}{b}$$

$$\therefore m_1(3m_1) = \frac{a}{b}$$

$$\therefore 3m_1^2 = \frac{a}{b}$$

$$\therefore m_1^2 = \frac{a}{3b} \quad \text{— (2)}$$

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from eq^n ① & ② $\Rightarrow$

$$\frac{h^2}{4b^2} = \frac{a}{3b}$$

$$\therefore \frac{h^2}{4b} = \frac{a}{3}$$

$$\therefore \frac{h^2}{ab} = \frac{4}{3}$$

16) If $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of lines, then $\lambda =$ ___

a) 4 b) 3 c) 2 d) 1

$\Rightarrow$ (c) Joint eq^n of lines is

$$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$$

Comparing this with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = \lambda$, $2h = -10$, $b = 12$, $2g = 5$, $2f = -16$, $c = -3$

$\therefore h = -5$ $\quad g = \frac{5}{2}$, $f = -8$

$\because$ Given eq^n represents pair of lines,

$\therefore abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\therefore \lambda(12)(-3) + 2(-8)\left(\frac{5}{2}\right)(-5) - \lambda(-8)^2 - 12\left(\frac{5}{2}\right)^2 - (-3)(-5)^2 = 0$$

$$\therefore -36\lambda + 200 - 64\lambda - 12\left(\frac{25}{4}\right) + 75 = 0$$

$$\therefore -100\lambda + 275 - 75 = 0$$

$$\therefore -100\lambda + 200 = 0$$

$$\therefore -100\lambda = -200$$

$$\therefore \lambda = 2$$

17) If θ is the angle between the lines $x^2 - 3xy + 2y^2 + \lambda x - 5y + 2 = 0$, then $\csc^2\theta =$ ___

a) 3 b) 9 c) 10 d) 100

$\Rightarrow$ (c) Comparing with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 1$, $2h = -3$, $b = 2$

$\therefore h = -\frac{3}{2}$

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$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{\frac{9}{4}-(1)(2)}}{1+2}\right|$$

$$= \left|\frac{2\sqrt{\frac{9-8}{4}}}{3}\right| = \left|\frac{2\times\frac{1}{2}}{3}\right| = \frac{1}{3}$$

$\therefore \cot\theta = 3$

Now, $\csc^2\theta = 1+\cot^2\theta = 1+9 = 10$

(18) If lines $px^2 - qxy - y^2 = 0$ make angles α & β with X-axis, then value of $\tan(\alpha+\beta) =$ ____

a) $\frac{-p}{1+q}$ b) $\frac{-q}{1+p}$ c) $\frac{q}{1+p}$ d) $\frac{p}{1+q}$

$\Rightarrow$ (b) Comparing given eq^n with $ax^2+2hxy+by^2=0$, we get

$$a = p,\ 2h = -q,\ b = -1$$

$$\therefore m_1 + m_2 = -\frac{2h}{b} = \frac{-(-q)}{-1} = -q$$

$$\& \ m_1 m_2 = \frac{a}{b} = \frac{p}{-1} = -p$$

$m_1 = \tan\alpha$ & $m_2 = \tan\beta$ .... Given

$$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

$$= \frac{m_1+m_2}{1-m_1\cdot m_2} = \frac{-q}{1-(-p)}$$

$$= \frac{-q}{1+p}$$

(19) If distance between lines $(x-2y)^2 + k(x-2y) = 0$ is 3 units, then $k =$ ____

a) ± 3 b) $\pm 5\sqrt{5}$ c) 0 d) $\pm 3\sqrt{5}$

$\Rightarrow$ (d) $(x-2y)^2 + k(x-2y) = 0$

$\therefore (x-2y)(x-2y+k) = 0$

$\therefore$ Separate eq^{ns} of lines are

$$x - 2y + 0 = 0$$

$$x - 2y + k = 0$$

$\therefore$ Lines are parallel & distance $= 3$ ... Give

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$\therefore \perp^{er}$ distance $= \dfrac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$

$$\therefore 3 = \frac{|0-k|}{\sqrt{1^2+(-2)^2}} = \frac{|-k|}{\sqrt{5}}$$

$$\therefore |k| = 3\sqrt{5}$$

$$\therefore k = \pm 3\sqrt{5}$$

20) Angle between lines
$(a^2-3b^2)x^2 + 8abxy + (b^2-3a^2)y^2 = 0$ is ____

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

$\Rightarrow$ (C) Joint eq^n of lines is
$(a^2-3b^2)x^2 + 8abxy + (b^2-3a^2)y^2 = 0$
Comparing with $Ax^2 + 2Hxy + By^2 = 0$, we get
$A = a^2-3b^2$, $2H = 8ab$, $B = b^2-3a^2$
$\therefore H = 4ab$

$$\therefore \tan\theta = \left|\frac{2\sqrt{H^2-AB}}{A+B}\right|$$

$$= \left|\frac{2\sqrt{16a^2b^2-(a^2-3b^2)(b^2-3a^2)}}{a^2-3b^2+b^2-3a^2}\right|$$

$$= \left|\frac{2\sqrt{16a^2b^2-(a^2b^2-3a^4-3b^4+9a^2b^2)}}{-2a^2-2b^2}\right|$$

$$= \left|\frac{2\sqrt{16a^2b^2-a^2b^2+3a^4+3b^4-9a^2b^2}}{-a^2-b^2}\right|$$

$$= \left|\frac{\sqrt{6a^2b^2+3a^4+3b^4}}{-a^2-b^2}\right|$$

$$= \left|\frac{\sqrt{3(a^2+b^2)^2}}{-(a^2+b^2)}\right| = \frac{\sqrt{3}(a^2+b^2)}{(a^2+b^2)} = \sqrt{3}$$

$$\therefore \tan\theta = \sqrt{3}$$

$$\therefore \theta = \frac{\pi}{3}$$

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